Wehrl information entropy and phase distributions of Schrödinger cat and cat-like states

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Abstract
The Wehrl information entropy and its phase density, the so-called Wehrl phase distribution, are applied to describe Schrödinger cat and cat-like (kitten) states. The advantages of the Wehrl phase distribution over the Wehrl entropy in a description of the superposition principle are presented. The entropic measures are compared with a conventional phase distribution from the Husimi Q-function. Compact-form formulae for the entropic measures are found for superpositions of well separated states. Examples of Schrödinger cats (including even, odd and Yurke–Stoler coherent states), as well as the cat-like states generated in the Kerr medium, are analysed in detail. It is shown that, in contrast to the Wehrl entropy, the Wehrl phase distribution properly distinguishes between different superpositions of unequally weighted states with respect to their number and phase-space configuration.

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1. Introduction

Schrödinger cats or cat-like states (kittens) are the striking manifestations of the superposition principle at the boundary between the quantum and classical regimes [1]. Especially since the 1980s, the Schrödinger cats have attracted much interest in quantum and atom optics or quantum computing by allowing controlled studies of quantum measurement, quantum entanglement and decoherence. Further interest has recently been triggered by first experimental demonstrations of Schrödinger cats on mesoscopic [2] and also macroscopic [3] scales.

We will analyse simple prototypes of the Schrödinger cat and cat-like states in entropic and phase descriptions. The Wehrl classical information entropy is defined to be [4]

$$S_{\mathrm{w}} \equiv - \int Q(\alpha) \ln Q(\alpha)\, \mathrm{d}^2\alpha \tag{1}$$

Table 1. Are the Wehrl information entropy, its density or Husimi PD good measures of the properties of classical and quantum fields?

| No	Measures	Wehrl entropy	Husimi PD	Wehrl PD
1	Discrimination of fields with random phase	yes	no	yes
2	Photon-number uncertainty	yes	no	yes
3	Discrimination of coherent states with different intensity	no	yes	yes
4	Phase locking	no	yes	yes
5	Phase bifurcation	no	yes	yes
6	Schrödinger cats with different weights	no	yes	yes
7	Schrödinger cat-like states with different weights	no	yes	yes

where $Q(\alpha)$ is the Husimi function given by $Q(\alpha) = \pi^{-1}\langle\alpha|\hat{\rho}|\alpha\rangle$ as the coherent-state representation of density matrix $\hat{\rho}$. The Wehrl classical information entropy, also referred to as the Shannon information of the Husimi Q-function, can be related to the von Neumann quantum entropy in different approaches [5,6], in particular in relation to a phase-space measurement [7]. It has been demonstrated that the Wehrl entropy is a useful measure of various quantum-field properties, including quantum noise [6–12], decoherence [9, 13], quantum interference [10], ionization [12], or squeezing [8, 14, 15]. Moreover, it has been shown that the Wehrl entropy gives a clear signature of the formation of the Schrödinger cat and cat-like states [11, 16] and a signature of splitting of the Q-function [13]. Here, we will show explicitly that the Schrödinger cat and cat-like states are, in general, *not* uniquely described by the conventional Wehrl entropy. Thus, for better description of Schrödinger cat and cat-like states, we apply another entropic measure—the so-called Wehrl phase distribution (Wehrl PD), defined to be the phase density of the Wehrl entropy [17]:

$$S_\theta \equiv -\int Q(\alpha)\ln Q(\alpha)|\alpha|\,\mathrm{d}|\alpha| \tag{2}$$

where $\theta = \mathrm{Arg}\alpha$. The Wehrl PD is simply related to the Wehrl entropy via integration, i.e. $S_\mathrm{w} = \int S_\theta\,\mathrm{d}\theta$. The Wehrl PD is formally similar to a conventional phase distribution from the Husimi Q-function, defined to be (see, e.g., [18])

$$P_\theta \equiv \int Q(\alpha)|\alpha|\,\mathrm{d}|\alpha| \tag{3}$$

which is referred to as the Husimi phase distribution (Husimi PD). The main physical advantage of the Wehrl PD over the conventional PDs (including that of Husimi) lies in its information-theoretic content. The Wehrl entropy, which is the area covered by the Wehrl PD, is a measure of information that takes into account the measuring apparatus (homodyne detection) used to obtain this information. Various other advantages of the Wehrl PD over the conventional PDs and the Wehrl entropy itself in a description of quantum and classical optical states of light were demonstrated in [17]. Several examples of applications of the Wehrl PD are listed in table 1. Example 1 is a consequence of the Wehrl-entropy sensitivity in discriminating different fields with random phase. By contrast, the conventional PDs are the same for arbitrary random-phase fields. Example 2 originates from the fact that the Werhl entropy is directly related to the phase-space measurement [7], thus in particular carrying information about the photon-number uncertainties. On the other hand, the conventional PDs do *not* measure photon-number properties. Examples 3–5, discussed in detail in [17], show advantages of both the Wehrl and conventional PDs over the Wehrl entropy. In the next sections we will compare descriptions of the Schrödinger cat and cat-like states in terms of the Wehrl entropy, and the Wehrl and Husimi
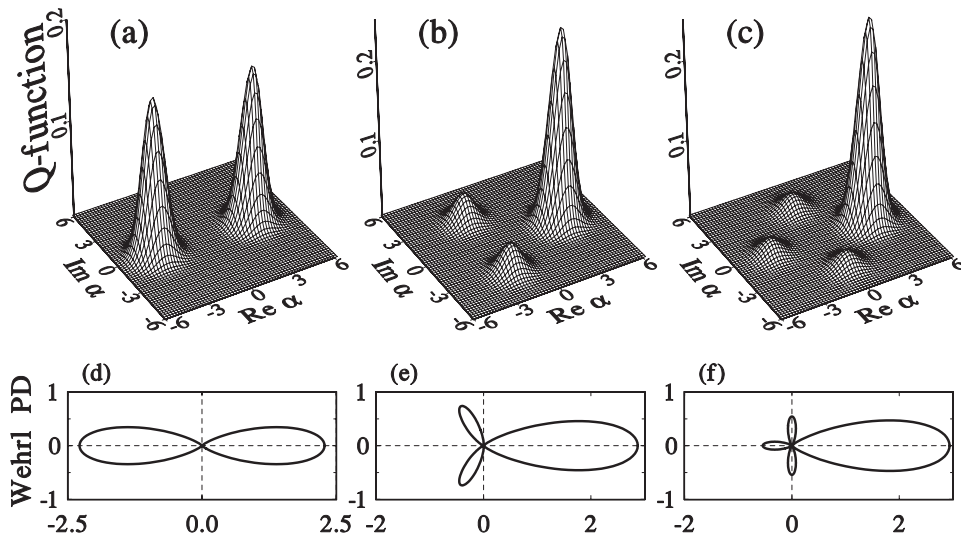
Figure 1. Equientropic Schrödinger cat and cat-like superpositions of $N = 2, 3, 4$ states. Husimi Q-functions (a)–(c) and the corresponding polar plots of Wehrl phase distributions (d)–(f) for well separated ($N < N_{\max}(\alpha_0 = \sqrt{12}) = 7$) superposition states described by the same Wehrl entropy.

PDs. Our analytical and numerical results are briefly summarized in table 1 by examples 6 and 7.

2. Equientropic cat and cat-like states

First, we will show that the conventional Wehrl entropy does *not* uniquely distinguish between different superpositions of unequally weighted states. Let us analyse in detail the following superposition of N coherent states:

$$|\overline{\alpha}_0\rangle_N = \mathcal{C}_N \left\{ \sqrt{1 - (N-1)x_N} |\alpha_0\rangle + \sqrt{x_N} \sum_{k=1}^{N-1} \left| \exp\left(ik\frac{2\pi}{N}\right)\alpha_0 \right\rangle \right\} \tag{4}$$

where $\mathcal{C}_N$ is the normalization constant and x_{N+1} are the roots of

$$f_{N+1}(x) = 2(1 - Nx)^{(1-Nx)} x^{Nx} - 1. \tag{5}$$

In particular equation (5) has the following roots: $x_2 = \frac{1}{2}$, $x_3 = 0.11\ldots$, $x_4 = 0.063\ldots$. For $N = 2$, the state $|\overline{\alpha}_0\rangle_2$ is the even coherent state, given by (10). We observe that the quantum superpositions $|\overline{\alpha}_0\rangle_N$ of well separated coherent states have the same Wehrl entropy, equal to

$$\lim_{(|\alpha_0|/N)\to\infty} S_w(|\overline{\alpha}_0\rangle_N) = 1 + \ln 2\pi \tag{6}$$

for any finite $N > 1$. The Q-functions for three different states with $N = 2, 3$ and 4 well separated components are depicted in figures 1(a)–(c). In the high-amplitude limit, the interference terms in the Q-function vanish and the normalization $\mathcal{C}_N$ becomes unity. The Wehrl entropy for these states is equal to $S_w = 1 + \ln 2\pi - \epsilon_N$ with the corrections (a) $\epsilon_2 < 0.000\,002$, (b) $\epsilon_3 < 0.000\,065$ and (c) $\epsilon_4 < 0.0012$ for the intensity $|\alpha_0|^2 = 12$. As proceeds from (6), these corrections can be made arbitrarily small by increasing $|\alpha_0|$ in comparison to N. In fact, for any quantum superposition of macroscopically distinguishable states, other superposition states can be found with the same Wehrl entropy but distinct number of components. Thus,

one can conclude that the conventional Wehrl entropy is not sensitive enough in discriminating cats from unequally weighted cat-like states. The main purpose of this paper is to apply a new entropic measure and to show its advantages over the conventional Wehrl entropy in describing macroscopically distinguishable superpositions of states.

3. New entropic description of cat and cat-like states

A standard example of the Schrödinger cat is a superposition of two coherent states $|\alpha_0\rangle$ and $|-\alpha_0\rangle$ in the form (see, e.g., [19])

$$|\alpha_0, \gamma\rangle = \mathcal{N}_\gamma \{|\alpha_0\rangle + \exp(\mathrm{i}\gamma)| - \alpha_0\rangle\} \tag{7}$$

with normalization

$$\mathcal{N}_\gamma = \left\{2\left[1 + \cos\gamma \exp(-2|\alpha_0|^2)\right]\right\}^{-1/2}. \tag{8}$$

For special choices of the superposition phase γ, the state $|\alpha_0, \gamma\rangle$ reduces to the well known Schrödinger cats, including the Yurke–Stoler coherent state for $\gamma = \pi/2$ [20]:

$$|\alpha_0\rangle_{\mathrm{YS}} = |\alpha_0, \pi/2\rangle = \frac{1}{\sqrt{2}} \left(|\alpha_0\rangle + \mathrm{i}| - \alpha_0\rangle\right) \tag{9}$$

and the even ($\gamma = 0$) and odd ($\gamma = \pi$) coherent states [21]:

$$|\alpha_0, 0\rangle = \mathcal{N}_0 \left(|\alpha_0\rangle + | - \alpha_0\rangle\right) = \frac{1}{\sqrt{\cosh|\alpha_0|^2}} \sum_{n=0}^{\infty} \frac{\alpha_0^{2n}}{\sqrt{(2n)!}} |2n\rangle \tag{10}$$

$$|\alpha_0, \pi\rangle = \mathcal{N}_\pi \left(|\alpha_0\rangle - | - \alpha_0\rangle\right) = \frac{1}{\sqrt{\sinh|\alpha_0|^2}} \sum_{n=0}^{\infty} \frac{\alpha_0^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle \tag{11}$$

respectively. The Husimi function for the Schrödinger cat $|\alpha_0, \gamma\rangle$ can be given in form of the sum

$$Q(\alpha) = \mathcal{N}_\gamma^2 [Q_1(\alpha) + 2Q_{12}(\alpha) + Q_2(\alpha)] \tag{12}$$

of the coherent components ($k = 1, 2$)

$$Q_k(\alpha) = \frac{1}{\pi} \exp\left\{ - \left|\alpha + (-1)^k \alpha_0\right|^2\right\} \tag{13}$$

and the interference term

$$Q_{12}(\alpha) = \frac{1}{\pi} \exp(-|\alpha|^2 - |\alpha_0|^2) \cos[\gamma + 2|\alpha| \, |\alpha_0| \sin(\theta_0 - \theta)] \tag{14}$$

where θ_0 is the phase of α_0. There is no compact-form exact expression of the phase distributions for the Schrödinger cat defined by (7). The states analysed in our former work [17] are among a few examples where the phase distributions can be expressed analytically in a compact form. Nevertheless, for well separated ($|\alpha_0| \gg 1$) coherent states $|\alpha_0\rangle$ and $| - \alpha_0\rangle$, we find that the Wehrl PD can be approximated by

$$S_\theta \approx \tfrac{1}{2}\left\{S_\theta^{\mathrm{cs}}(\alpha_0) + S_\theta^{\mathrm{cs}}(-\alpha_0) + \ln 2\left[P_\theta^{\mathrm{cs}}(\alpha_0) + P_\theta^{\mathrm{cs}}(-\alpha_0)\right]\right\} \tag{15}$$

in terms of the Werhl and Husimi phase distributions for coherent states [17]:

$$S_\theta^{\mathrm{cs}}(\alpha_0) = \frac{1}{2\pi} \mathrm{e}^{X^2 - X_0^2} \left\{\mathrm{e}^{-X^2} f_2 + \sqrt{\pi} X \left[1 + \mathrm{erf}(X)\right] f_1\right\} \tag{16}$$

$$P_\theta^{\mathrm{cs}}(\alpha_0) = \frac{1}{2\pi} \mathrm{e}^{X^2 - X_0^2} \left\{\mathrm{e}^{-X^2} + \sqrt{\pi} X \left[1 + \mathrm{erf}(X)\right]\right\} \tag{17}$$

respectively, where

$$f_j = X_0^2 - X^2 + \ln \pi + j/2$$
$$X = |\alpha_0| \cos(\theta - \theta_0)$$

(18)

and $X_0 = X(\theta = \theta_0) = |\alpha_0|$. Moreover, $\mathrm{erf}(X)$ is the error function. As was discussed in [17], the Wehrl and Husimi PDs for coherent states differ by the factors f_j only. The Q-function and Wehrl PD for the cat $|\alpha_0, \gamma\rangle$ with well separated components ($|\alpha_0| \gg 1$) are presented in figures 1(a) and (d), respectively. In this case, the contribution of the interference term $Q_{12}(\alpha)$ is negligible. Thus, the Q-function and Wehrl PD for $|\alpha_0|^2 = 12$ are practically independent of the superposition coefficient γ, and the difference between the exact Wehrl PD and its approximation, given by (15), vanishes. This can be understood better by analysing figure 2, where the Wehrl and Husimi PDs are depicted for different values of separation amplitude α_0 for three superposition parameters γ corresponding to the Yurke–Stoler, even and odd coherent states. It is apparent, in both the Wehrl and Husimi PDs, that the differences among cats described by (9)–(11) diminish with increasing amplitude α_0 and would completely disappear even at $|\alpha_0| = 2.4$ on the scale of figure 2. It is also seen that the maximum values of P_θ and S_θ depend on the separation amplitude $|\alpha_0|$. However, in the case of the Wehrl PD only, the area under the curve is amplitude dependent, being an indicator of the phase-space uncertainty. Despite the formal similarities, the Wehrl and Husimi PDs differ recognizably for superposition of states which are not well separated ($|\alpha_0| < 1$). For example, the Wehrl PD for the even coherent state is less than that for the odd cat for all phases θ at small values of separation parameter ($|\alpha_0| \leqslant 0.8$). The physical interpretation of this behaviour can be given as follows. The contribution of the vacuum (single-photon) state is dominant for the even (odd) coherent state with small separation parameter. According to the entropic analysis of quantum noise [17], the Wehrl PD for vacuum is smaller than that for the single-photon state for all phases θ. This implies that $S_\theta^{\mathrm{even}} < S_\theta^{\mathrm{odd}}$ for $|\alpha_0| \ll 1$. In contrast, this inequality does not hold for the corresponding Husimi PDs: P_θ^{even} can be less but also greater than P_θ^{odd} for some values of phase θ at $|\alpha_0| \leqslant 0.8$. The Wehrl PD for the Yurke–Stoler coherent state, S_θ^{YS}, approaches S_θ^{odd} for $\theta < \pi$ and S_θ^{even} for $\theta > \pi$, as the best presented in figure 2 for $|\alpha_0| = 1.2$. In comparison, the Husimi PD P_θ^{YS} differs more significantly from P_θ^{even} and P_θ^{odd} than the corresponding Wehrl PDs for the same $|\alpha_0| < 2.4$.

The Wehrl entropy for the cat, given by (7), has been studied numerically by Bužek *et al* [10] for arbitrary superposition phase γ, whereas Jex and Orłowski [16] and Vaccaro and Orłowski [11] studied the Wehrl entropy for the Yurke–Stoler coherent state generated in a Kerr-like medium. In figure 3, we show the Wehrl entropies S_{w} for the cat $|\alpha_0, \gamma\rangle$ in dependence on the superposition phase γ for various values of the separation amplitude α_0. The curve for $\alpha_0 = 0.8$ corresponds to the case analysed by Bužek *et al* [10]. The discrepancies between Wehrl entropies for the even, odd and Yurke–Stoler cats vanish with increasing $|\alpha_0|$. The Wehrl entropy in the high-amplitude limit ($|\alpha_0| \gg 1$) can be approximated by

$$S_{\mathrm{w}} \approx 1 + \ln \pi - |c_1|^2 \ln |c_1|^2 - |c_2|^2 \ln |c_2|^2$$

(19)

which for superpositions of equal-amplitude states reduces to $S_{\mathrm{w}} \approx 1 + \ln(2\pi)$. This value can be obtained by integrating the approximate Wehrl PD, given by (15). On the scale of figure 3, the curve representing the Wehrl entropy for $\alpha_0 = 2.4$ is practically indistinguishable from the entropy in the infinite-amplitude limit.

The Schrödinger cat-like state, also referred to as the kitten state[5], is a generalization of the Schrödinger cat for a macroscopically distinct superposition state with more than two

[5] The term 'Schrödinger's kitten' in the above sense was coined by Agarwal *et al* [22]. In contrast, some authors (e.g. Taubes [23]) prefer to use this term to refer to a small (mesoscopic) Schrödinger cat, which should be macroscopic according to Schrödinger's original idea [1].
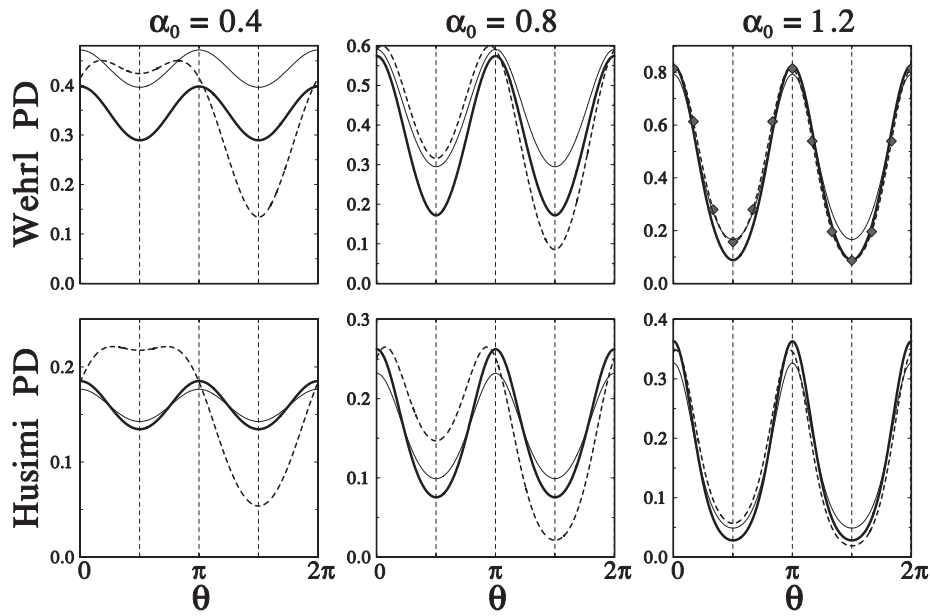
Figure 2. Wehrl and Husimi phase distributions for three types of Schrödinger cat: even (thick solid curves), odd (thin solid curves) and Yurke–Stoler (dashed curves or those with diamonds) coherent states for different values of the coherent amplitude α_0.
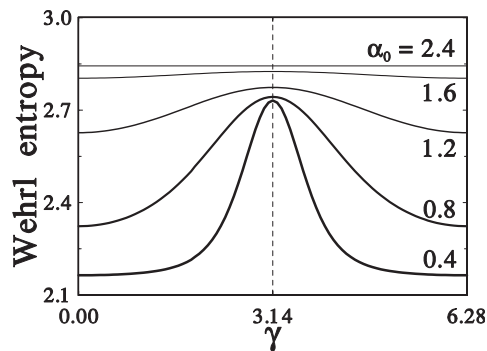
Figure 3. Wehrl entropy for Schrödinger cat $|\alpha_0, \gamma\rangle$ in dependence on the superposition phase γ for different values of the coherent amplitude α_0.

components. In particular the normalized superposition of N coherent states

$$|\alpha_0\rangle_N = \sum_{k=1}^{N} c_k |\exp(\mathrm{i}\phi_k)\alpha_0\rangle \tag{20}$$

is the standard example of the Schrödinger cat for $N = 2$ and the Schrödinger kitten for $N > 2$. Equation (20) is valid for arbitrary number, amplitudes and phases of the states in the superposition. The state defined by (7) is a special case of (20) for two coherent states with opposite phases ($\phi_2 - \phi_1 = \pi$) and $c_2/c_1 = \exp(\mathrm{i}\gamma)$. The Husimi Q-function for the Schrödinger cat-like state reads as [19, 24]

$$Q(\alpha) = Q_0(\alpha) + Q_{\mathrm{int}}(\alpha) = \sum_{k=1}^{N} |c_k|^2 \, Q_k(\alpha) + 2 \sum_{k>l} |c_k||c_l| \, Q_{kl}(\alpha) \tag{21}$$

where the free part, $Q_0(\alpha)$, is the sum of the coherent terms

$$Q_k(\alpha) = \frac{1}{\pi} \exp\left\{-\left|\alpha - e^{i\phi_k}\alpha_0\right|^2\right\} \tag{22}$$

and the interference part, $Q_{\text{int}}(\alpha)$, is given in terms of

$$Q_{kl}(\alpha) = \sqrt{Q_k Q_l} \cos\left[\gamma_k - \gamma_l + 2|\alpha|\,|\alpha_0|\cos(\phi_{kl}^{(+)} + \theta_0 - \theta)\sin\phi_{kl}^{(-)}\right]. \tag{23}$$

The phases in (23) are defined as $\gamma_k = \text{Arg}\,c_k$, $\theta = \text{Arg}\,\alpha$, $\theta_0 = \text{Arg}\,\alpha_0$ and $\phi_{kl}^{(\pm)} = \frac{1}{2}(\phi_k \pm \phi_l)$, where ϕ_k appears in (20). The Husimi Q-function, given by (21), is a generalization of (12) for arbitrary number of components in the superposition state. The compact-form exact analytical expressions exist neither for the phase distributions nor the Wehrl entropy. However, for well separated (i.e. if $|\alpha_0| \gg N$) states, the Wehrl PD for the Schrödinger cat-like state, given by (20), is approximately equal to

$$S_\theta \approx \sum_{k=1}^{N} |c_k|^2 S_\theta^{\text{cs}}(e^{i\phi_k}\alpha_0) - \sum_{k=1}^{N} |c_k|^2 \ln(|c_k|^2) P_\theta^{\text{cs}}(e^{i\phi_k}\alpha_0). \tag{24}$$

The Wehrl PDs for the equientropic high-intensity states, given by (4), were calculated from (24) and presented in figures 1(d)–(f) in comparison to their Husimi Q-representations given in figures 1(a)–(c), respectively. The Wehrl PDs clearly show the number, amplitude and phase-space configuration of the Schrödinger cat and cat-like states. We emphasize that the curves in figures 1(d)–(f) cover approximately the same area, equal to $S_{\text{w}} \approx 1 + \ln 2\pi$. Equation (24) for the equal-amplitude superposition goes over into

$$S_\theta \approx \frac{1}{N} \sum_{k=1}^{N} \left\{S_\theta^{\text{cs}}(e^{i\phi_k}\alpha_0) + P_\theta^{\text{cs}}(e^{i\phi_k}\alpha_0) \ln N\right\} \tag{25}$$

where $S_\theta^{\text{cs}}(\exp\{i\phi_k\}\alpha_0, 0)$ are the coherent-field Wehrl PDs given by (16) and $P_\theta^{\text{cs}}(\exp\{i\phi_k\}\alpha_0, 0)$ are the coherent-field Husimi PDs described by (17). In (25), we have assumed that the superposition coefficients are the same for all components of the cat-like state, i.e. $c_k = \text{const} = 1/\sqrt{N}$. Equation (24) leads, after integration over θ, to the Wehrl entropy

$$S_{\text{w}} \approx 1 + \ln \pi - \sum_{k=1}^{N} |c_k|^2 \ln |c_k|^2. \tag{26}$$

In the special case of equally weighted superposition states, (26) simplifies to $S_{\text{w}} \approx 1 + \ln(N\pi)$ in agreement with the result of Jex and Orłowski [16]. Estimations of the maximum number, N_{max}, of well separated states in the superposition (20) as a function of amplitude $|\alpha_0|$ can be given, for example, by [24]

$$N_{\text{max}}(\alpha_0) = \text{Int}(2^{-1/2}\pi|\alpha_0|) \tag{27}$$

where $\text{Int}(x)$ is the integer part of x. Approximations (24)–(26) are valid for $N \leqslant N_{\text{max}}(\alpha_0)$.

Several methods have been proposed to generate Schrödinger's cat or cat-like states (see [2, 3], and [19] for a review). In particular it has been predicted that a coherent light propagating in a Kerr-like medium, described by the Hamiltonian

$$\hat{H} = -\frac{1}{2}\hbar g \hat{a}^{\dagger 2} \hat{a}^2 \tag{28}$$

where g is the coupling constant, can be transformed into the Schrödinger cat (Yurke–Stoler coherent state) [20] and kittens [24,25] at some evolution times. Explicitly, for $gt = 2\pi\frac{M}{N}$, where
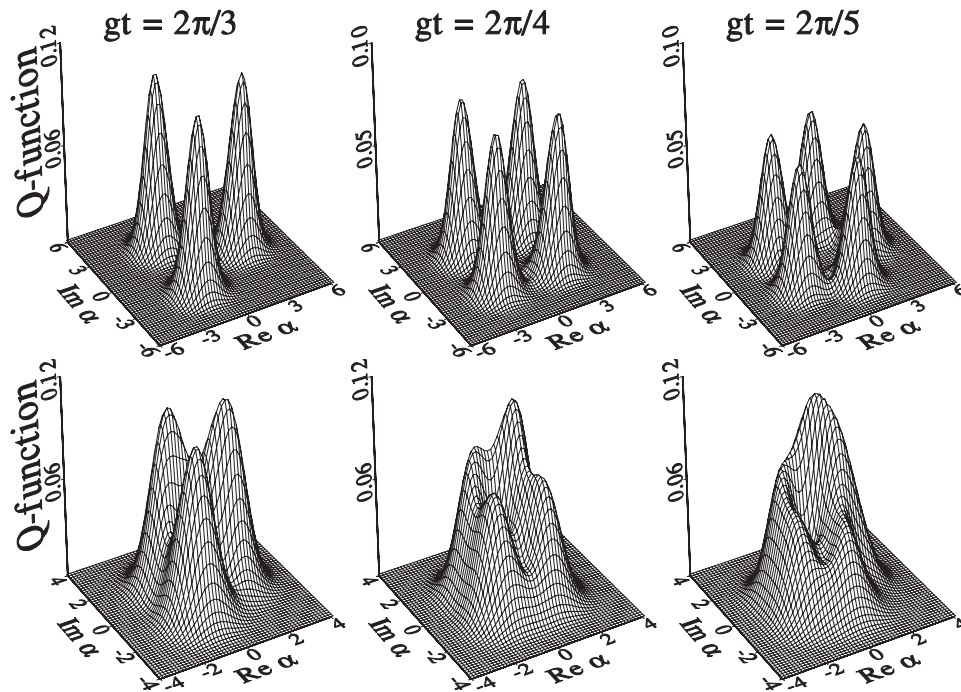
Figure 4. Husimi Q-function for Schrödinger cat-like states generated in Kerr medium at different evolution times $gt = 2\pi/N$ ($N = 3, 4, 5$) for initial coherent states with amplitudes $\alpha_0 = 3$ (first row) and $\alpha_0 = \sqrt{2}$ (second row).

M and N are mutually prime numbers, the generated state is a superposition of N coherent states, given by (20), with the phases $\phi_k = (2k + N - 3)\frac{\pi}{N}$ and superposition coefficients

$$c_k = \frac{1}{N} \sum_{n=1}^{N} \exp\left\{ in\left[\frac{M}{N}\pi(n-1) - \phi_k \right] \right\}. \tag{29}$$

Jex and Orłowski [16] and Vaccaro and Orłowski [11] have shown, by analysing the model described by (28), that the Wehrl entropy gives a clear signature of the formation of the Schrödinger cat-like states. The Wehrl PD, in comparison to the Wehrl entropy, offers more detailed description of superposition states, showing explicitly the phase configuration and amplitudes of the components. The Husimi Q-function in figure 4, and the Wehrl and Husimi PDs in figure 5, are presented for the Schrödinger cat-like states generated by Hamiltonian (28) for different evolution times gt in two regimes determined by the initial amplitudes: $N < N_{\max}(3) = 6$ and $N \geqslant N_{\max}(\sqrt{2}) = 3$, where $N = 3, 4, 5$. The number N of well separated peaks in S_θ and P_θ clearly corresponds to the number of states in the superposition. The analysis of the Wehrl PD shows how the amplitude of the incident coherent beam determines the maximum number of readily distinguishable states. For example, both the Husimi and Wehrl PDs depicted by thin curves in figure 5 have regular and readily distinguishable structures even for five-component superposition of the initial amplitude $|\alpha_0| = 3$. However, the four- and five-component superpositions for the initial condition $|\alpha_0| = \sqrt{2}$ are highly deformed, as plotted by the thick curves in figure 5. Thus, the Wehrl and Husimi PDs describe the influence of the interference terms (23) on formation of the Schrödinger cat-like states. Distributions P_θ and S_θ are similar. However, a closer analysis reveals their differences. In particular, as seen
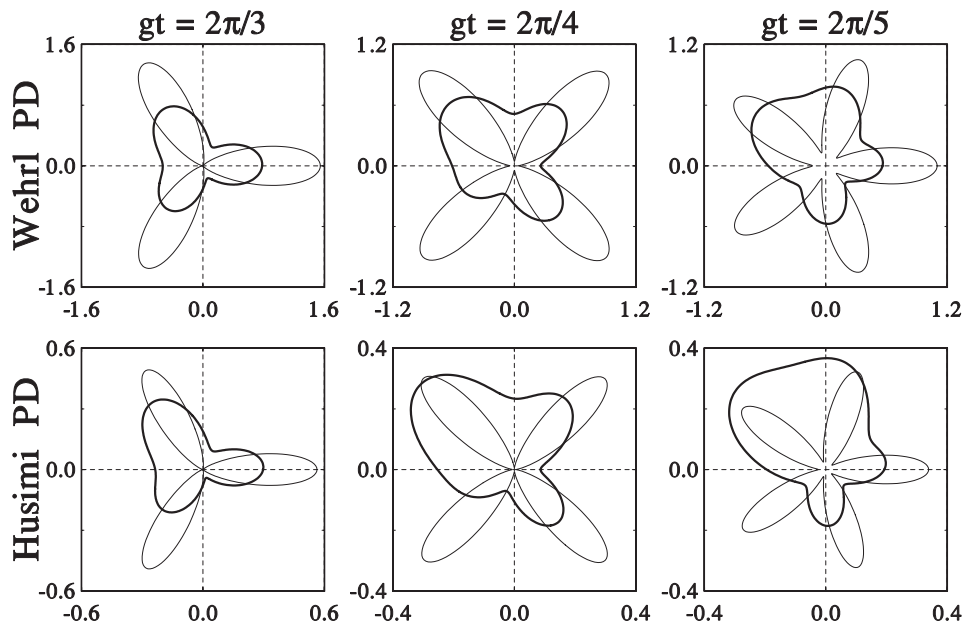
Figure 5. Polar plots of Wehrl and Husimi phase distributions for the same Schrödinger cat-like states as those in figure 4: thin curves correspond to $\alpha_0 = 3$ and thick curves to $\alpha_0 = \sqrt{2}$.

in figure 5 for well separated states, the rosette leaves are broader for S_θ than for P_θ. For states not separated distinctly, S_θ exhibits slightly more regular behaviour than P_θ. The influence of interference terms is more pronounced for S_θ, as can be concluded by analysing the thin curves in the range close to zero for $gt = 2\pi/5$ or $gt = 2\pi/4$. Nevertheless, the main advantage of our description in terms of S_θ over that of P_θ resides in the area covered by S_θ, which is equal to the Wehrl entropy. Thus a simple phase-space operational interpretation can be applied [7].

4. Conclusions

The purpose of the paper was to find a good information-theoretic measure of the superposition principle. We have shown that the conventional Wehrl entropy is, in general, *not* a good measure for discriminating two-component (Schrödinger cats) from multi-component (Schrödinger kittens) macroscopically distinct superposition states. We have applied a new information measure—the Wehrl phase distribution, which is a phase density of the Wehrl entropy [17]. Compact form estimations of both Wehrl measures were found for the superpositions of well separated states. It was demonstrated that the Wehrl phase distribution, in contrast to the Wehrl entropy, properly distinguishes the number and phase-space configuration of the Schrödinger cat and cat-like states even with unequally weighted components.

Acknowledgments

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